Topology

Semestral Examination

Instructions: All questions carry ten marks.

1. Define Hausdorff space. Let X be a topological space and $A \subset X$ be a subspace. Let $f : A \to Z$ be a continuous map into a Hausdorff space Z. Show that there is at most one extension of f to a continuous function $g : \bar{A} \to Z$.

2. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ that is continuous at exactly one point of $\mathbb{R}$. Justify your answer.

3. A map $f : X \to Y$ is called *locally constant* if each point x of X has an open neighbourhood such that the restriction of f to that neighbourhoos is a constant map.

   (a) Show that any locally constant map is continuous.

   (b) Prove that a locally constant map on a connected set is a constant map.

4. Define compact space. Prove that any compact subset of a Haudorff space is closed.

5. Define regular space and normal space. Show that any regular second countable space is normal.

6. Show that every locally compact Hausdorff space is regular.